

Automated frequency band estimation for sinusoidal low-pass signals in noise

A numerical study on synthetic and natural signals

Jakob Harden (Graz University of Technology)

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Classic signal analysis usually begins with collecting information about the properties of the signal to the greatest extent possible. This information is the basis for the sensible selection of analysis methods and their parameterization.

For sinusoidal signals, the frequency band of the signal is one of these fundamental signal properties. Estimating the frequency band can be a challenge, especially for signals with a small signal-to-noise ratio.

This work proposes a fully automated method for easily, quickly, and stably estimating the frequency band of low-pass signals. The estimation is based on the discrete Fourier transform and threshold detection. Due to the complete automation, special attention is paid to the method's stability.

The performance of the method is tested here using synthetic and natural signals. The preliminary results show that the automated estimation and visual detection deliver comparable results for the upper frequency band limit.

► WHAT

- Estimate upper frequency band limit for damped sinusoidal low-pass signals.

► WHY

- Gain fundamental signal information for the subsequent signal analysis.

► HOW

- Estimate power spectral density of the signal (PSD).
- Threshold detection using the cumulative sum of the PSD.

► USAGE

- Synthetic and natural low-pass signals (page 9 ff).
- Signals from ultrasonic pulse transmission tests (page 17 ff).

► HIGHLIGHT

- **Automated estimation and visual inspection deliver comparable results.**

► Materials

- Synthetic signals: damped sinusoidal low-pass signals in noise (for testing purposes)
- Natural signals from ultrasonic pulse transmission tests (cement paste, air, water)

► Methods

- Power spectral density (PSD) estimation
- Cumulative sum
- Threshold detection

► Assumptions

- signals are (mostly) mean-free, (damped) sinusoidal low-pass signals
- uncorrelated i.i.d. noise, Gaussian white noise

► Synthetic test signals - damped sinusoidal signals in noise

- Signal model $x_{LP} = s_{LP} + \nu$
- $s_{LP}[n] = A \cdot \left(\sum_{F=1}^5 \sin(2\pi F \frac{n}{N-1}) \right) \cdot e^{-DF \frac{n}{N-1}}$
- $n = 0, \dots, N-1, \quad N = F_s \cdot N_{cy}$
- Noise ν , i.i.d. noise, Gaussian white noise
- Signal frequencies $F = (1, 2, 3, 4, 5)$ Hz, sampling frequency $F_s = 100$ Hz
- Amplitude $A = 1$ V, number of cycles $N_{cy} = 3$, exponential decay factor DF
- Noise floor before and behind the middle signal section, $L_{pre} = L_{post} = F_s$

► Analysis settings:

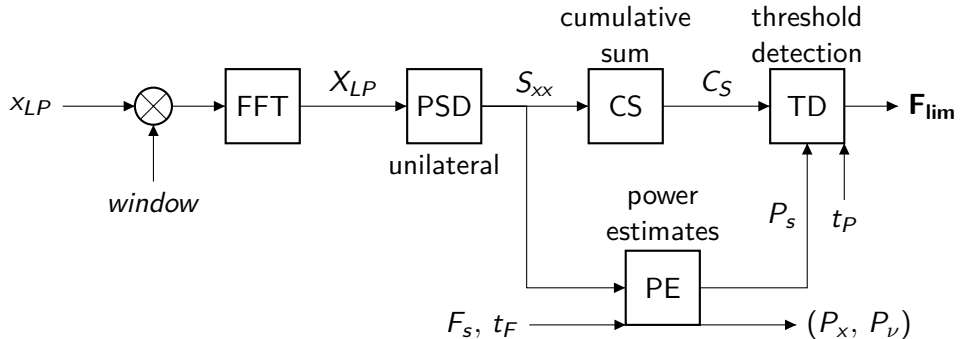
- Frequency threshold factor $t_F = 0.75$ (noise measurement range: 25 to 100 Hz)
- Power threshold factor $t_P = 0.93$
- No FFT zero-padding

► Signal series from ultrasonic pulse transmission tests

- Specimen: cement paste, water-cement-ratio $w/c = 0.4$
- Ultrasonic measuring distance: $D = 50$ mm
- Piezo-electric sensor's natural frequency: $F_{nat} = 500$ kHz
- Pulse voltage: 800 V, Pulse width: $2.5 \mu\text{Seconds}$
- Sampling frequency: $F_s = 10$ MHz
- Vertical resolution: 16 bit
- Test duration: 24 hr (cement paste tests)
- Recording interval: 5 Min (cement paste tests)
- Two-channel test: primary wave (P-wave) and secondary wave (S-wave)

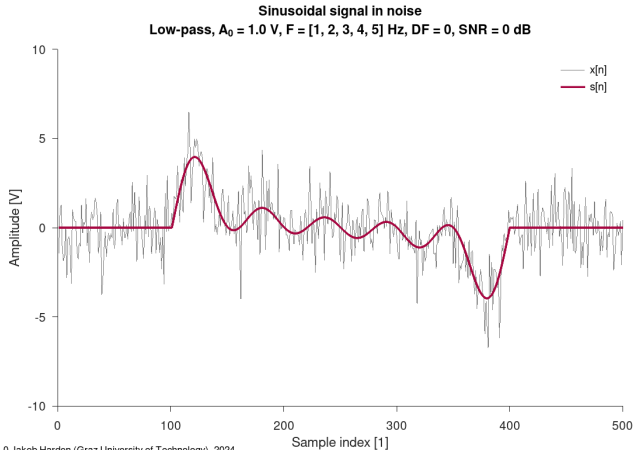
► Analysis settings:

- Frequency threshold factor: $t_F = 0.9$ (PSD noise measurement range: 1 to 10 MHz)
- Power threshold factor: $t_P = 0.93$
- FFT zero-padding: 3 times signal length, $N_{FFT} = 4 N$



x_{LP} ... low-pass signal, F_s ... sampling frequency, t_F ... frequency threshold factor, t_P ... power threshold factor, P_x ... power of signal in noise, P_ν ... noise power, P_s ... signal power, F_{lim} ... estimate upper frequency band limit. See also notes on next page.

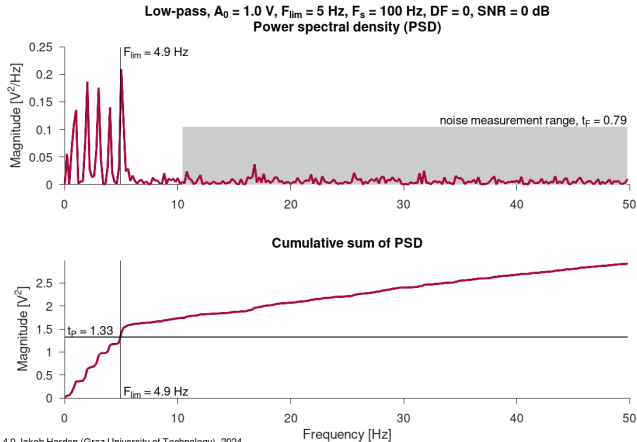
- ▶ Frequency threshold factor t_F
 - multiplier of the Nyquist frequency $F_{ny} = F_s/2$
 - used to determine the frequency range to estimate the noise power P_ν
 - can be obtained from initial visual inspection and/or measurement device characteristics
- ▶ Power threshold factor t_P
 - multiplier of the estimate signal power $P_s = P_x - P_\nu$
 - avoid detection over-run for signals with a high signal-to-noise ratio ($P_\nu \approx 0$)
- ▶ PSD scaling: magnitudes are scaled such that the sum of the magnitudes is equal to the total signal power $P_x = \sum S_{xx}$.
- ▶ Cumulative sum (CS) characteristics considering low-pass signals
 - Frequency 0 to F_{lim} : steep curve until approx. signal energy P_s is reached
 - Frequency F_{lim} to F_{ny} : mostly linear increasing curve (slope depends on P_ν)
- ▶ Algorithm: see Appendix, page 29



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Test signal 1:

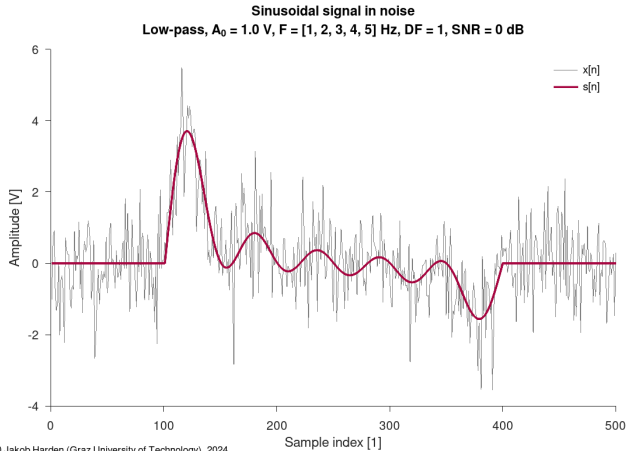
- ▶ no exponential decay, $DF = 0$
- ▶ low signal-to-noise ratio, $SNR = 0$ dB



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Test signal 1, results:

- ▶ CS: sharp transition at F_{lim}
- ▶ F_{lim} is slightly under-estimated
- ▶ error ≈ -0.1 Hz $\equiv$ 0.2 % of F_{ny}

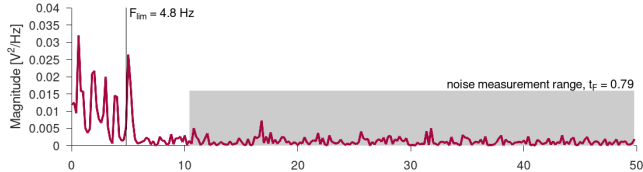


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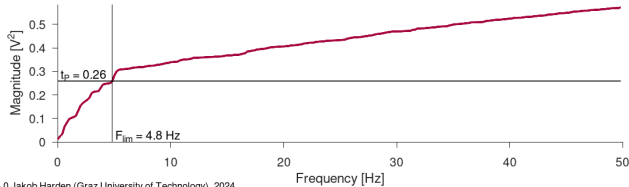
Test signal 2:

- ▶ low exponential decay, $DF = 1$
- ▶ low signal-to-noise ratio, $SNR = 0$ dB

Low-pass, $A_0 = 1.0$ V, $F_{lim} = 5$ Hz, $F_s = 100$ Hz, $DF = 1$, $SNR = 0$ dB
Power spectral density (PSD)



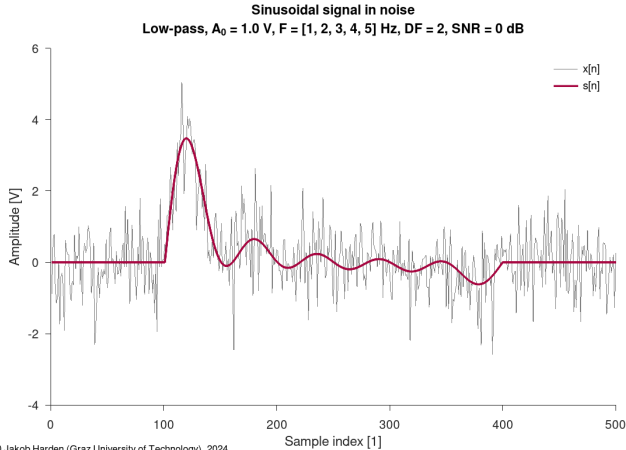
Cumulative sum of PSD



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Test signal 2, results:

- ▶ CS: sharp transition at F_{lim}
- ▶ F_{lim} is slightly under-estimated
- ▶ error ≈ -0.2 Hz $\equiv$ 0.4 % of F_{ny}

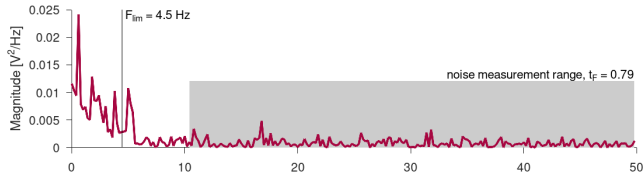


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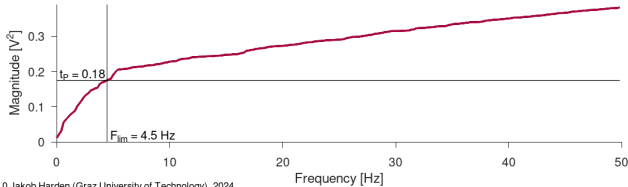
Test signal 3:

- moderate exponential decay, $DF = 2$
- low signal-to-noise ratio, $SNR = 0$ dB

Low-pass, $A_0 = 1.0$ V, $F_{lim} = 5$ Hz, $F_s = 100$ Hz, $DF = 2$, $SNR = 0$ dB
Power spectral density (PSD)



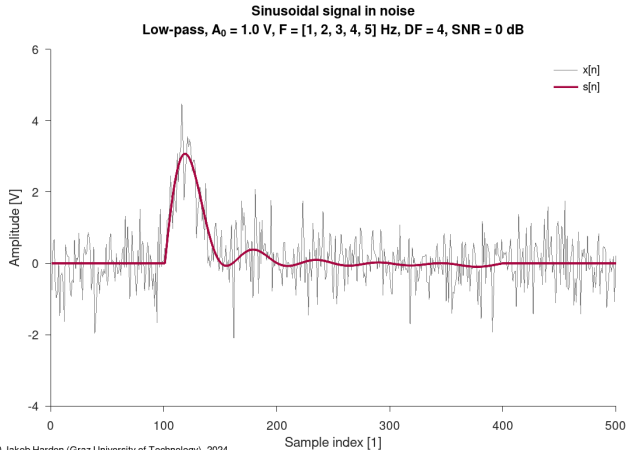
Cumulative sum of PSD



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Test signal 3, results:

- ▶ CS: sharp transition at F_{lim}
- ▶ F_{lim} is slightly under-estimated
- ▶ error ≈ -0.5 Hz $\equiv$ 1.0 % of F_{ny}

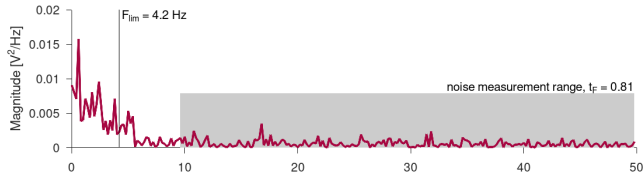


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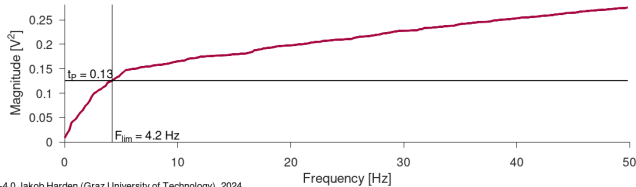
Test signal 4:

- ▶ strong exponential decay, $DF = 4$
- ▶ low signal-to-noise ratio, $SNR = 0$ dB

Low-pass, $A_0 = 1.0$ V, $F_{lim} = 5$ Hz, $F_s = 100$ Hz, $DF = 4$, $SNR = 0$ dB
Power spectral density (PSD)



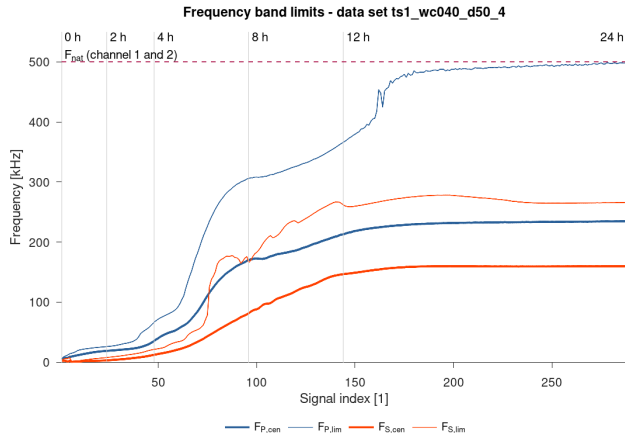
Cumulative sum of PSD



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Test signal 4, results:

- ▶ CS: smooth transition at F_{lim}
- ▶ F_{lim} is slightly under-estimated
- ▶ error ≈ -0.8 Hz $\equiv$ 1.6 % of F_{ny}



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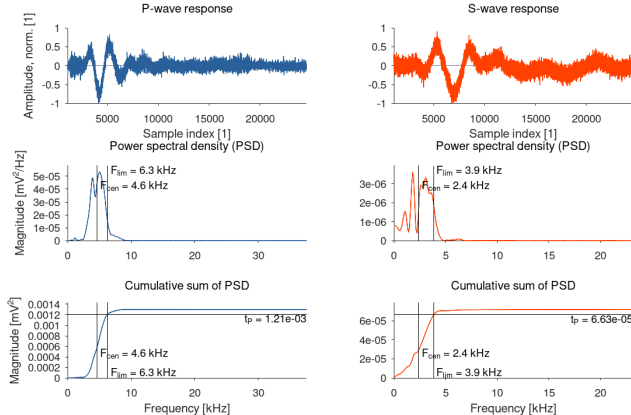
Data set:

- ▶ ts1_wc040_d50_4 [3]
- ▶ 288 signals per channel

Observations:

- ▶ P-wave: maximum $F_{lim} \approx 100\%$ of F_{nat}
- ▶ S-wave: maximum $F_{lim} \approx 55\%$ of F_{nat}

Data set ts1_wc040_d50_4, SID = 1

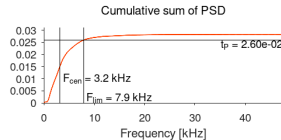
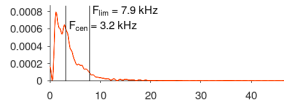
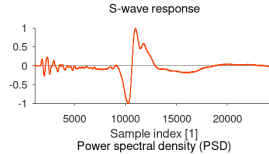
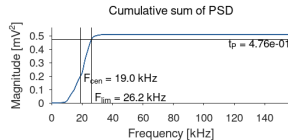
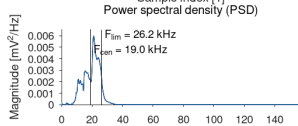
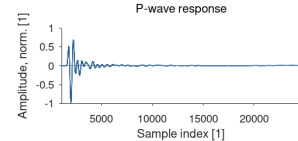


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Data set:

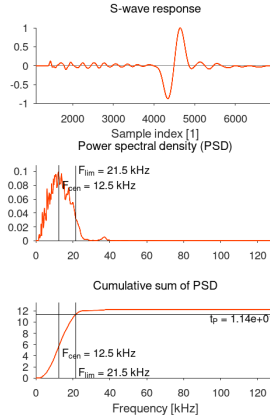
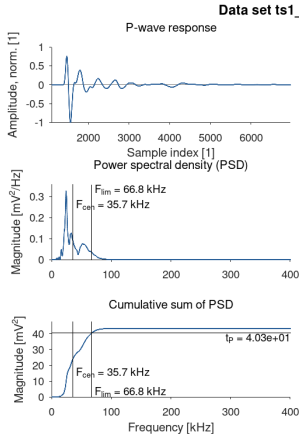
- ▶ ts1_wc040_d50_4 [3]
- ▶ signal #1 of 288, maturity 0 hr
- ▶ **Note:** the center frequency F_{cen} is the weighted average of all frequencies in the detected frequency band

Data set ts1_wc040_d50_4, SID = 24



Data set:

- ▶ ts1_wc040_d50_4 [3]
- ▶ signal #24 of 288, maturity 2 hr



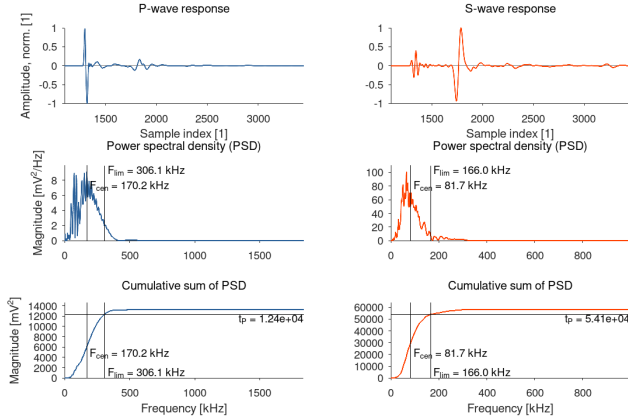
Data set:

- ▶ ts1_wc040_d50_4 [3]
- ▶ signal #48 of 288, maturity 4 hr

Data set:

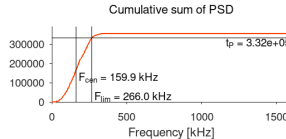
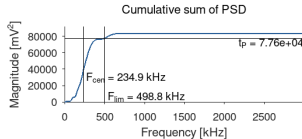
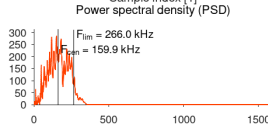
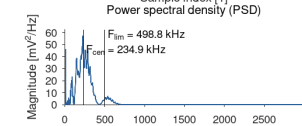
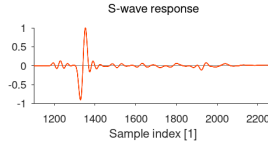
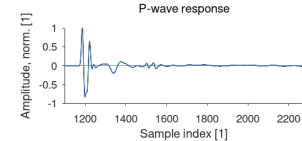
- ▶ ts1_wc040_d50_4 [3]
- ▶ signal #96 of 288, maturity 8 hr

Data set ts1_wc040_d50_4, SID = 96



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Data set ts1_wc040_d50_4, SID = 288



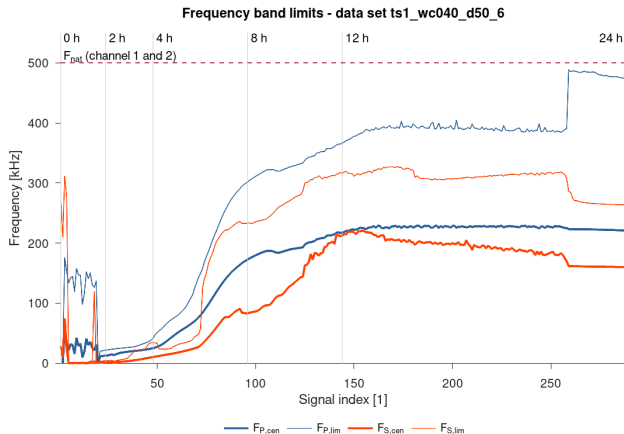
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Data set:

- ▶ ts1_wc040_d50_4 [3]
- ▶ signal #288 of 288, maturity 24 hr

Observations:

- ▶ P-wave PSD: additional side lobe at $F \approx 500 \text{ kHz}$



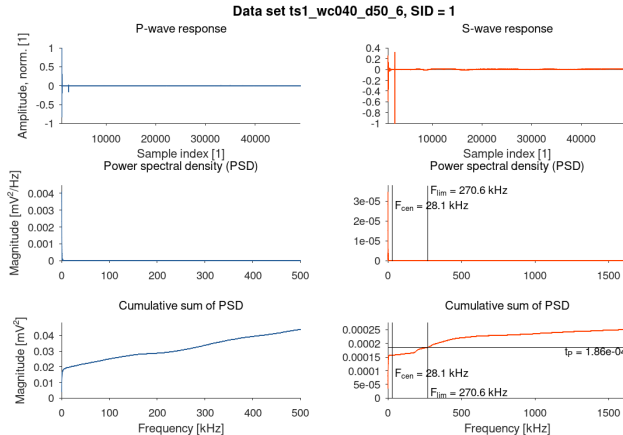
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Data set:

- ▶ ts1_wc040_d50_6 [3]
- ▶ 288 signals per channel

Observations:

- ▶ P-wave, S-wave:
increased F_{lim} at low maturity where low frequency limits are expected (maturity: 0 to 2 hr)



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Data set:

- ▶ ts1_wc040_d50_6 [3]
- ▶ signal #1 of 288, maturity 0 hr

Observations:

- ▶ P-wave PSD: vanishing signal, detection failed!
- ▶ S-wave PSD: F_{lim} is overestimated significantly!

In the Appendix, additional analysis results for the datasets listed below are shown.

- ▶ Cement paste tests[3], $w/c = 0.4$, $D = 25$ mm, page 34
- ▶ Cement paste tests[3], $w/c = 0.4$, $D = 70$ mm, page 36
- ▶ Reference tests, air[4], $D = 25$ mm, page 38
- ▶ Reference tests, air[4], $D = 50$ mm, page 40
- ▶ Reference tests, air[4], $D = 70$ mm, page 42
- ▶ Reference tests, water[5], $D = 25$ mm, page 44
- ▶ Reference tests, water[5], $D = 50$ mm, page 46
- ▶ Reference tests, water[5], $D = 70$ mm, page 48

With respect to the chosen analysis parameters, the results presented before allow for the following conclusions:

► Synthetic signals

- In the cumulative sum, the transition at the frequency limit F_{lim} becomes smoother with increasing decay factor. This smoothness is related to the exponential decay and spectral leakage. It also has an impact on the threshold detection.
- The frequency limit is generally slightly underestimated.
- The proposed method delivers usable frequency limit estimates even when noise is dominant ($\text{SNR} = 0 \text{ dB}$) and therefore threshold detection in the PSD becomes difficult.

► Natural signals

- The method also shows good stability with natural signals and their broad bandwidth regarding frequencies, noise and decay. The counter example on page 23 shows that the method fails if the signal power becomes very low.
- For the vast majority of signals the frequency band limit F_{lim} is limited by the sensor's natural frequency.
- In the signal series of the cement paste tests, additional side lobes in the power spectrum can occasionally be observed in the range of the sensor's natural frequency and above. This indicates that the sensor can oscillate freely at its natural frequency. However, the possibility that the test specimen is at least partially decoupled from the sensor surface cannot be ruled out.

Aside: The computation of an entire series of about 600 signals takes about one second.

Now that the method is described in its basic principles and first results are available, further questions still arise.

- ▶ How do estimation results depend on the spectral leakage of the discrete Fourier transformation? Spectral leakage arises from exponential decay and non-coherent sampling.
- ▶ How does the amount of signal data influence estimation accuracy and precision? Power estimates, especially the noise power, depends on the amount of available data.
- ▶ How do the results change when changing the method's estimation schema towards accuracy? Yet, the noise power in the PSD's low-pass section is not taken into account.
- ▶ How to extend the proposed method to band-pass signals? Yet, the method is designed for low-pass signals only.

Note: To support the future development of the proposed method, the supplementary GNU Octave code[1] and the $\text{\LaTeX}$ code[2] of this presentation is made available publicly under open licenses.

- [1] Jakob Harden. *Automated frequency band estimation for sinusoidal low-pass signals in noise - supplementary GNU Octave code*. Version 1.0.0. Dec. 2024. DOI: 10.3217/bry0f-cb052. URL: <https://doi.org/10.3217/bry0f-cb052>.
- [2] Jakob Harden. *Automated frequency band estimation for sinusoidal low-pass signals in noise - supplementary LaTeX code*. Version 1.0.0. Dec. 2024. DOI: 10.3217/zm80p-90g35. URL: <https://doi.org/10.3217/zm80p-90g35>.

- [3] Jakob Harden. *Ultrasonic Pulse Transmission Tests: Datasets - Test Series 1, Cement Paste at Early Stages*. Version 1.0. Graz University of Technology, July 2023. DOI: 10.3217/bhs4g-m3z76. URL: <https://doi.org/10.3217/bhs4g-m3z76>.
- [4] Jakob Harden. *Ultrasonic Pulse Transmission Tests: Datasets - Test Series 5, Reference Tests on Air*. Version 1.0. Graz University of Technology, July 2023. DOI: 10.3217/bjkrj-pg829. URL: <https://doi.org/10.3217/bjkrj-pg829>.
- [5] Jakob Harden. *Ultrasonic Pulse Transmission Tests: Datasets - Test Series 6, Reference Tests on Water*. Version 1.0. Graz University of Technology, July 2023. DOI: 10.3217/hn7we-q7z09. URL: <https://doi.org/10.3217/hn7we-q7z09>.

$\mathbf{x} \in \mathbb{R}^N$... signal amplitude array

$F_s \in \mathbb{R}$... sampling frequency

$z \in \mathbb{R}$... zero-padding factor, multiplier of the signal's length N

$F_{lim} = \text{ESTIMATE_FREQUENCY_BAND}(\mathbf{x}, F_s, z, t_P, t_F)$

Ensure: $0.8 \leq t_P \leq 0.99$, $t_P \in \mathbb{R}$

▷ signal power threshold factor

Ensure: $0.5 \leq t_F \leq 0.9$, $t_F \in \mathbb{R}$

▷ frequency threshold factor

1: $\mathbf{F}, \mathbf{S}_{xx} \leftarrow \text{PSD}(\mathbf{x}, F_s, \lfloor Nz \rfloor)$ ▷ unilateral power spectral density (PSD) and frequencies

2: $N_{bin} \leftarrow |\mathbf{F}|$ ▷ number of frequency bins

3: $L_\nu \leftarrow \lfloor N_{bin} t_F \rfloor$ ▷ length of noise measurement section

4: $L_s \leftarrow N_{bin} - L_\nu$ ▷ length of signal section

Continued on next page.

- 5: $P_x \leftarrow \frac{1}{N} \sum_{n=0}^N x_n^2$ ▷ estimate power of signal in noise
- 6: $P_\nu \leftarrow \frac{N_{bin}}{L_\nu} \sum_{k=L_s}^{N_{bin}-1} S_{xx,k}$ ▷ estimate noise power
- 7: $P_s \leftarrow P_x - P_\nu$ ▷ estimate signal power
- 8: $C_k \leftarrow \sum_{m=0}^k S_{xx,k}, k = 0, \dots, N_{bin} - 1$ ▷ cumulative sum of the PSD
- 9: $U_m \leftarrow k, \forall (C_k \neq C_j, k \neq j), k, j = 0, \dots, N_{bin} - 1$ ▷ bin indices for unique values of $\mathbf{C}$
- 10: $\mathbf{x}_{int} \leftarrow C_k, k \in U$ ▷ unique sequence of interpolation supports
- 11: $\mathbf{y}_{int} \leftarrow F_k, k \in U$ ▷ sequence of interpolation support amplitudes
- 12: $F_{lim} \leftarrow INTERP1(\mathbf{x}_{int}, \mathbf{y}_{int}, P_s t_P)$ ▷ piece-wise, linear interpolation

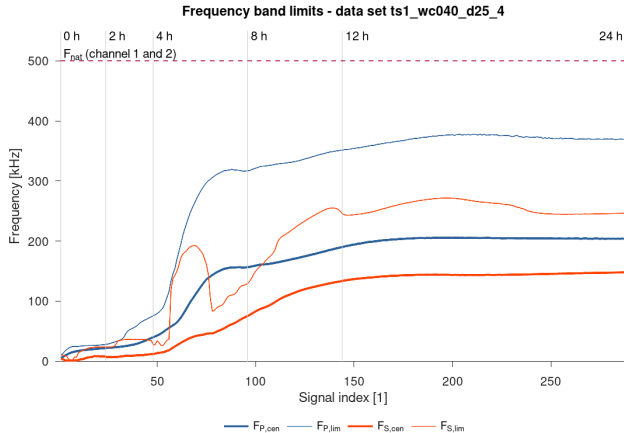
Continued on next page.

- 13: $k_{lim} \leftarrow \inf(\arg(\mathbf{F} \geq F_{lim}))$
 - 14: $L_s \leftarrow \min(2 k_{lim}, \lceil 0.5 N_{bin} \rceil)$
 - 15: $L_\nu \leftarrow N_{bin} - L_s$
 - 16: $P_\nu \leftarrow \frac{N_{bin}}{L_\nu} \sum_{k=L_s}^{N_{bin}-1} S_{xx,k}$
 - 17: $P_s \leftarrow P_x - P_\nu$
 - 18: $F_{lim} \leftarrow INTERP1(\mathbf{x}_{int}, \mathbf{y}_{int}, P_s t_P)$
 - 19: **return** F_{lim}
- ▷ find bin index close to frequency limit
 - ▷ update length of signal section
 - ▷ update length of noise measurement section
 - ▷ update estimate noise power
 - ▷ update estimate signal power
 - ▷ piece-wise, linear interpolation
 - ▷ return estimate frequency band limit

Function PSD , see program code[1], function file `tool_est_dft.m`

Function $INTERP1$, see GNU Octave function documentation `interp1`

Function $ESTIMATE_FREQUENCY_BAND$, see program code, function file `tool_est_fqband.m`



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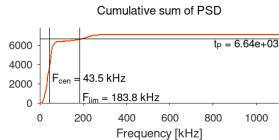
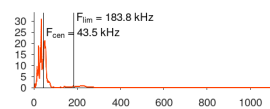
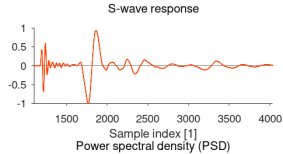
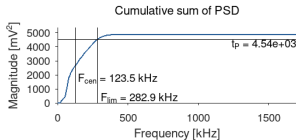
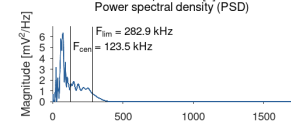
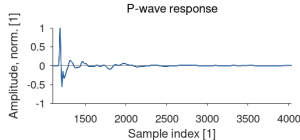
Data set:

- ▶ ts1_wc040_d25_4 [3]
- ▶ 288 signals per channel

Observations:

- ▶ S-wave: 'hill' around maturity of 5.5 hr

Data set ts1_wc040_d25_4, SID = 72

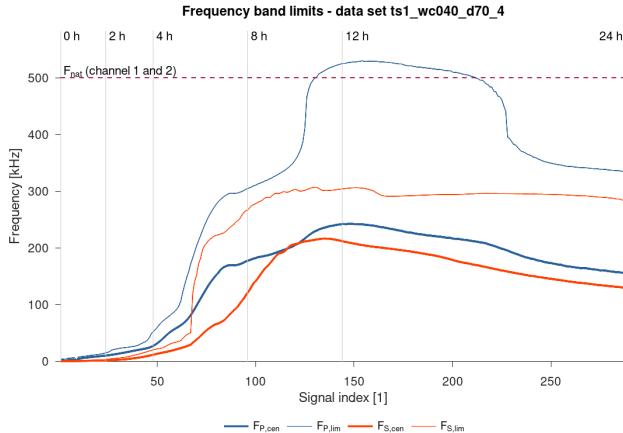


Data set:

- ▶ ts1_wc040_d25_4 [3]
- ▶ signal #72 of 288, maturity = 6 hr

Observations:

- ▶ S-wave PSD: small side lobe at $F \approx 250$ kHz



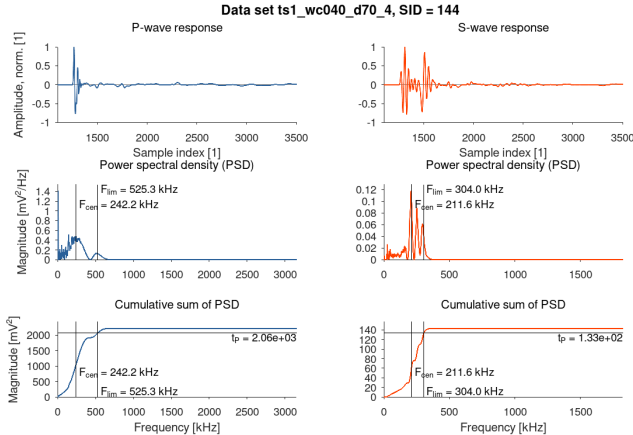
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Data set:

- ▶ ts1_wc040_d70_4 [3]
- ▶ 288 signals per channel

Observations:

- ▶ P-wave: maximum $F_{lim} \approx 110\%$ of F_{nat} , 'hill' between 10 and 18 hr
- ▶ S-wave: maximum $F_{lim} \approx 60\%$ of F_{nat}

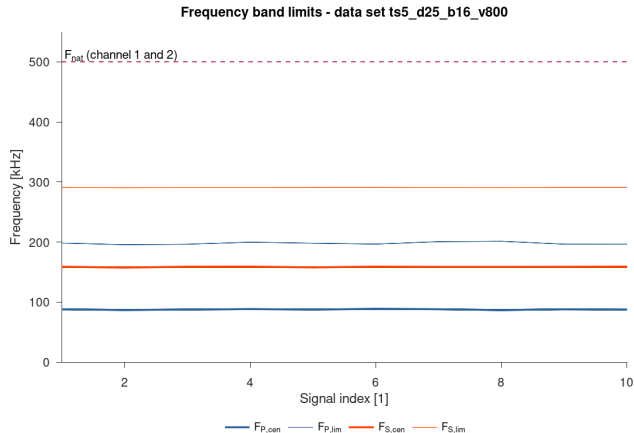


Data set:

- ▶ ts1_wc040_d70_4 [3]
- ▶ signal #144 of 288, maturity = 12 hr

Observations:

- ▶ P-wave PSD: additional side lobe at $F \approx 525$ kHz



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Data set:

- ▶ ts5_d25_b16_v800 [4]
- ▶ 10 repetitions

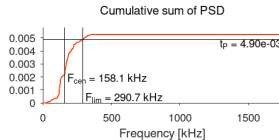
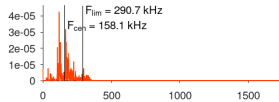
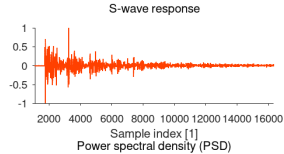
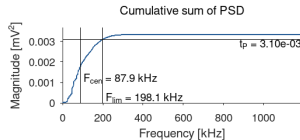
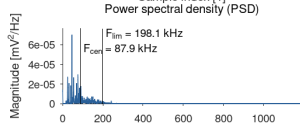
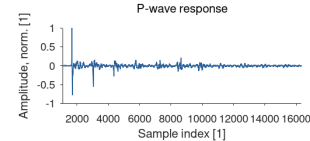
Observations:

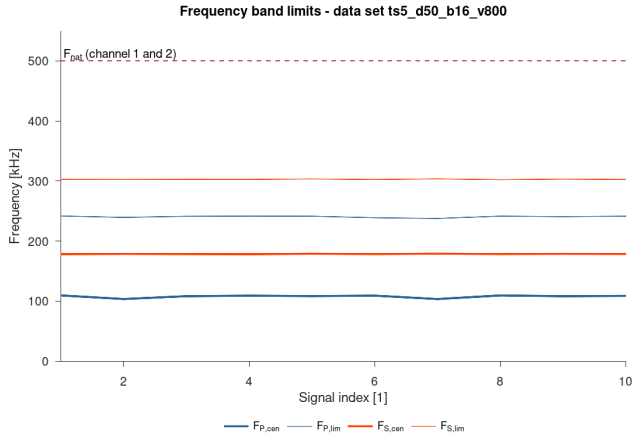
- ▶ P-wave: $F_{lim} \approx 40$ % of F_{nat}
- ▶ S-wave: $F_{lim} \approx 60$ % of F_{nat}

Data set:

- ▶ ts5_d25_b16_v800 [4]
- ▶ Signal #5 of 10 repetitions

Data set ts5_d25_b16_v800, SID = 5





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Data set:

- ▶ ts5_d50_b16_v800 [4]
- ▶ 10 repetitions

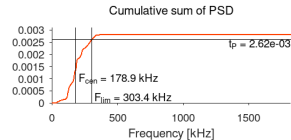
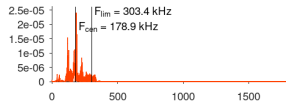
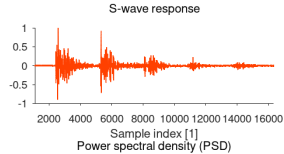
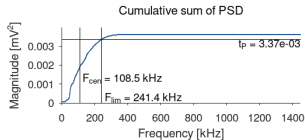
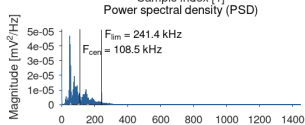
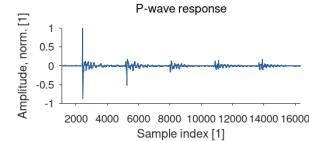
Observations:

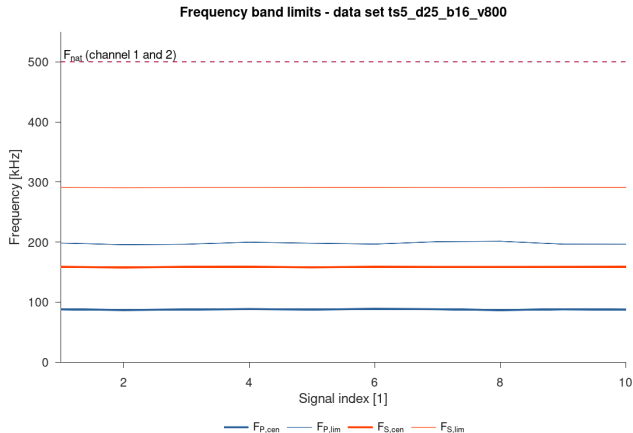
- ▶ P-wave: $F_{lim} \approx 50$ % of F_{nat}
- ▶ S-wave: $F_{lim} \approx 60$ % of F_{nat}

Data set:

- ▶ ts5_d50_b16_v800 [4]
- ▶ Signal #5 of 10 repetitions

Data set ts5_d50_b16_v800, SID = 5





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Data set:

- ▶ ts5_d70_b16_v800 [4]
- ▶ 10 repetitions

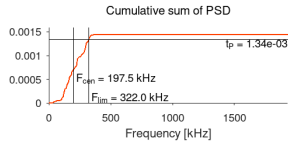
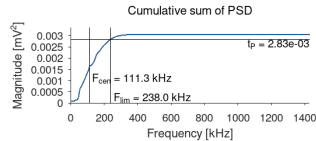
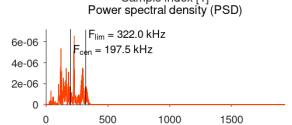
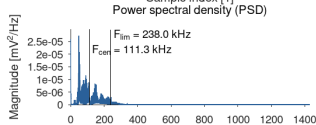
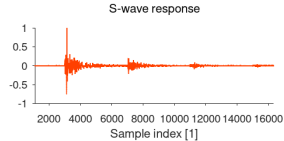
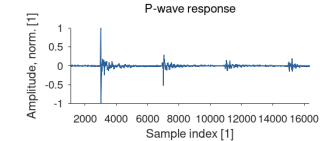
Observations:

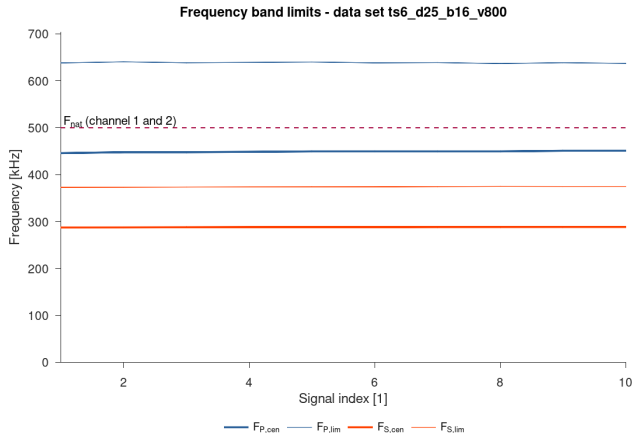
- ▶ P-wave: $F_{lim} \approx 40$ % of F_{nat}
- ▶ S-wave: $F_{lim} \approx 65$ % of F_{nat}

Data set:

- ▶ ts5_d70_b16_v800 [4]
- ▶ Signal #5 of 10 repetitions

Data set ts5_d70_b16_v800, SID = 5





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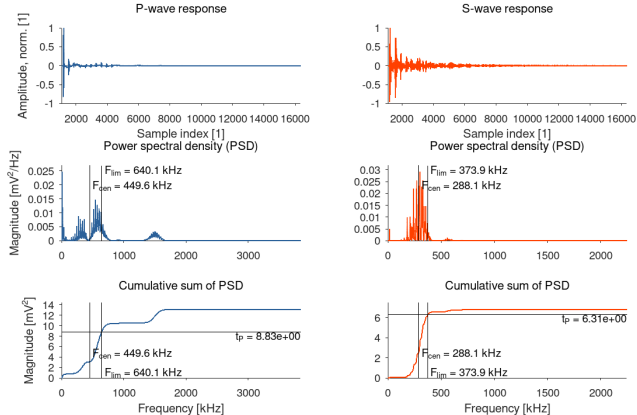
Data set:

- ▶ ts6_d25_b16_v800 [5]
- ▶ 10 repetitions

Observations:

- ▶ P-wave: $F_{lim} \approx 130$ % of F_{nat}
- ▶ S-wave: $F_{lim} \approx 75$ % of F_{nat}

Data set ts6_d25_b16_v800, SID = 5



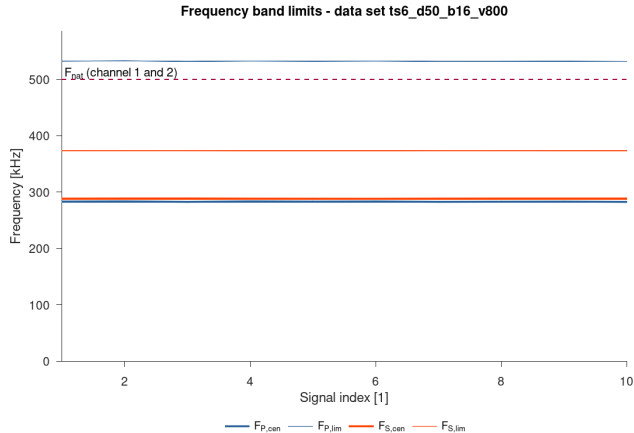
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Data set:

- ▶ ts6_d25_b16_v800 [5]
- ▶ Signal #5 of 10 repetitions

Observations:

- ▶ P-wave PSD: additional side lobes at F_{nat} and $3 F_{nat}$, F_{lim} is too high



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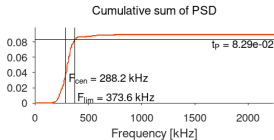
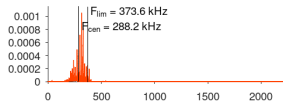
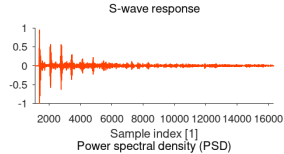
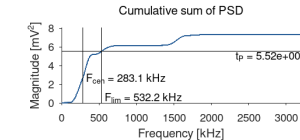
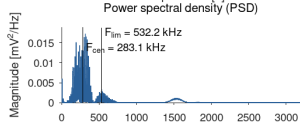
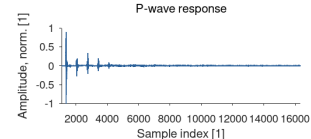
Data set:

- ▶ ts6_d50_b16_v800 [5]
- ▶ 10 repetitions

Observations:

- ▶ P-wave: $F_{lim} \approx 110$ % of F_{nat}
- ▶ S-wave: $F_{lim} \approx 75$ % of F_{nat}

Data set ts6_d50_b16_v800, SID = 5

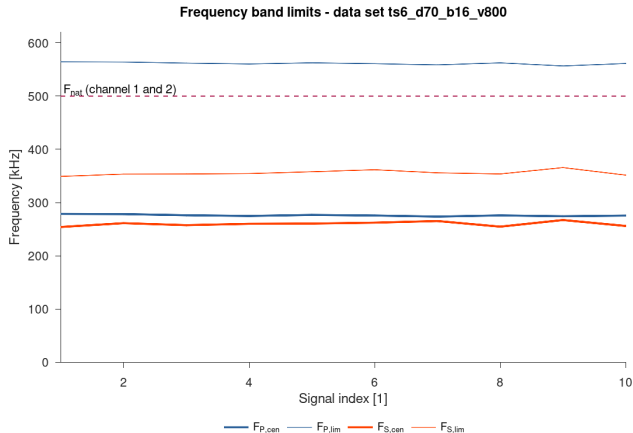


Data set:

- ▶ ts6_d50_b16_v800 [5]
- ▶ Signal #5 of 10 repetitions

Observations:

- ▶ P-wave PSD: additional side lobes at F_{nat} and $3 F_{\text{nat}}$, F_{lim} is too high



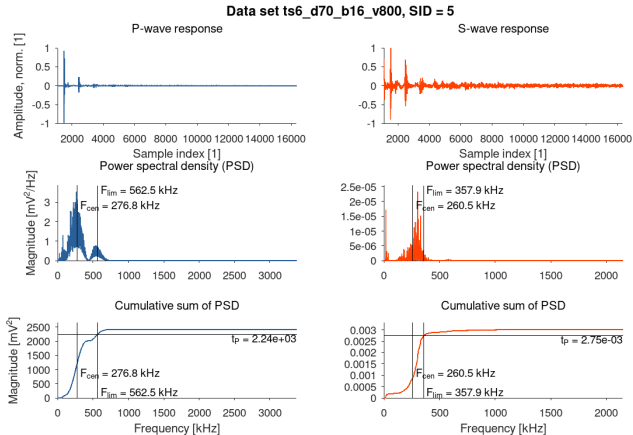
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Data set:

- ▶ ts6_d70_b16_v800 [5]
- ▶ 10 repetitions

Observations:

- ▶ P-wave: $F_{lim} \approx 112$ % of F_{nat}
- ▶ S-wave: $F_{lim} \approx 75$ % of F_{nat}



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Data set:

- ▶ ts6_d70_b16_v800 [5]
- ▶ Signal #5 of 10 repetitions

Observations:

- ▶ P-wave PSD:
additional side lobe at F_{nat} , F_{lim} is too high

First name Jakob
Last name Harden
Affiliation Graz University of Technology, Graz, Austria
Website jakobharden.at
Email jakob.harden@tugraz.at, jakob.harden@student.tugraz.at, office@jakobharden.at
ORCID 0000-0002-5752-1785
LinkedIn [jakobharden](https://www.linkedin.com/in/jakobharden)



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